Recitation 9/24 Recitation

Feel free to use Julia to help with row reduction throughout this worksheet!

1. Consider the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 12 \\ 2 \end{bmatrix} \right\}$.

   (a) Determine whether the set is linearly independent.

   (b) Find a basis for $\text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 12 \\ 2 \end{bmatrix} \right\}$. What is its dimension?

2. Let $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ -1 & -3 & 7 & 8 \\ 5 & 2 & 1 & 6 \end{bmatrix}$. Find a basis for the column space and null space of A.

3. In class, we saw (in some examples) that a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of vectors in $\mathbb{R}^n$ is linearly dependent if and only if (at least) one of the vectors is a linear combination of the others. Let's prove this fact in general.

   (a) Show that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent, then one of the vectors is a linear combination of the others.

   (b) Show that if one of the vectors in $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a linear combination of the others, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent.

4. **Looking Ahead** Let A be an $n \times n$ matrix. Explain why the following statements are equivalent.

   (a) A is invertible

   (b) $N(A) = \{\mathbf{0}\}$

   (c) The columns of A are linearly independent